A Mathematical Modeling Project Using MATLAB®

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Mathematical software has been playing an important role in teaching and learning. This paper reports the students' performance and reaction on the use of MATLAB® for a modeling project at the National Taiwan Normal University. It was the first time a mathematical modeling project had been incorporated into the Applied Mathematics course for such students at this University. It was also the first time the students experiencing applied mathematical modeling as well as using the mathematical software. The outcomes of the project were encouraging and the approach of using MATLAB® seems to be an effective method in learning mathematics.

1. Introduction

In the traditional education in Taiwan, students are taught the steps of doing mathematics. The traditional learning strategies such as memorizing and practicing the standard type of problems are prevalent in Taiwan classrooms. Most teachers and students are also under tremendous pressure at school; the students struggle to obtain good examination results in order to enter the best universities and the teachers' performance is determined by their students' results. The learning situation makes it difficult for students to appreciate mathematics.

Why mathematical modeling? In mathematical modeling activities, students will gain the situational reasoning, mathematizing, interpreting and communication (Lin *et al.*, 2005). The purpose to including this modeling project in the course of Applied Mathematics was to give students some practical experience in mathematics and reinforce the usefulness of mathematics in the 'real world'. The primary intention of the modeling project was to make the learning of Applied Mathematics more interesting as it modeled what was, for them, a 'real life' problem.

Figure 1 is a typical cycle of the process of mathematical modeling (Clatwothy, 1989). At the beginning of the modeling (Figure 1), the problem is specified and certain assumptions about the problem are made, follows by formulating model. To solve the model is to solve the problem; to

interpret the solution of the model is to understand the causes. However, the solution to the model may not always match the situation as it actually exists, in which case the model must be reformulated. Mathematical modeling can be a long process if the problem is a complex one. Though the process can be tedious, it imparts valuable skills and experiences to students, especially when they 'interpret the solution'. Students are often asked to solve mathematics problems but they rarely need to interpret the solutions or to explain the solutions.
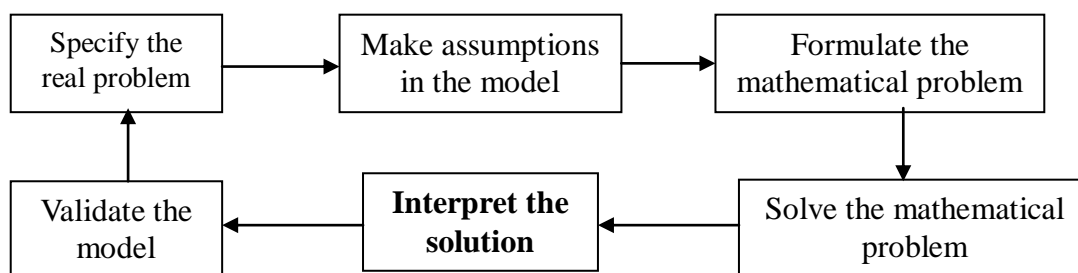

Figure 1. Mathematical modeling cycle.

Why mathematical software? In today's century, computing has been an essential skill for workforce development. It not only gets work done efficiently, it also often helps to present work explicitly. Hence it is important that students acquire the skill during their schooldays. In order to present mathematical solution explicitly, picture is usually the best for it as a picture is worth a thousand words. The mathematical software, MATLAB$^{®}$ was chosen to use in this modeling project because it was the software provided by the university and it also produces good graphics.

The twenty-six students taking the Applied Mathematics course were all Taiwanese. The majority were in their second year of university, though there were four fourth year students and nine third year students. It was the first time any of the students took an applied mathematical modeling project as well as used the mathematical software, MATLAB$^{®}$. The Applied Mathematics was a one-semester course and the modeling project was taken place during the course.

Here, we report mainly on the findings of using MATLAB$^{®}$ in mathematical modeling, some findings about the students' attitudes in doing the modeling toward mathematics can be found in Lim *et al.* (2007).

2. Framework

In order to understand how the students felt about doing applied mathematical modeling using mathematical software, the following outlines were set as our study guide:

- What are students' understandings about the application of mathematics in real world?
- What are students' views on the usefulness of mathematics?
- Are students excited to explore more results from the solution?

In this mathematical modeling project, students were required to present their findings from the project (using PowerPoint slides) in class. The presentations were to help determine their feelings and difficulties in doing the project as well as in using the mathematical software.

In this study, we also looked into the attitudes which were beliefs, usefulness, enjoyment and anxiety from the perspective of applied mathematical modeling project. But these are reported in Lim *et al.* (2007), hence we will not discuss them again in this paper.

3. The project

3.1 The model

The project was modeling of volcanic ash fall after an eruption. This was a research work of the first author and she was also the instructor of the class. The mathematical model was the advection-dispersion equation, a partial differential equation. The advection-dispersion model (Lim *et al.*, 2005):

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} + V\frac{\partial c}{\partial y} - S\frac{\partial c}{\partial z} - D_x\frac{\partial^2 c}{\partial x^2} - D_y\frac{\partial^2 c}{\partial y^2} = Q\delta(t)\delta(x-X)\delta(y-Y)\delta(z-H)$$

where c is the concentration; Q is the total mass of release, U and V are the wind speed in x and y directions respectively; S is the falling speed of particle; D_x and D_y are the dispersion in x and y directions respectively; t is the time; x, y and z are coordinate directions; X, Y and H the coordinates of the release point. The assumptions given to the model were $c = 0$ at $t^- = 0$ and $c \to 0$ as $x \to \pm\infty$, $y \to \pm\infty$, $z \to +\infty$ and z=0.

From the model, the solution of the deposit on the ground was derived as

$$\frac{SQ}{4\pi H\sqrt{D_x D_y}}e^{-\frac{\left(x-X-U\frac{H}{S}\right)^2}{4D_x\frac{H}{S}}-\frac{\left(y-Y-V\frac{H}{S}\right)^2}{4D_y\frac{H}{S}}}$$

(1)

3.2 Students' tasks

Students were only required to interpret the parameters of the solution within the context of the model. Due to the time constraints, the mathematical modeling project only emphasized on 'interpreting the solution' (see Figure 1). In order to interpret the solution, students may vary the values of the parameters to interpret the plot produced by MATLAB®. Interpretation of the solution is important as applied mathematical modeling usually deals with real life problems and it is important that (mathematics) students are able to interpret the solutions to those whose expertise is not in mathematics. For example, in this project, modeling of volcanic ash fall, volcanologists may not be keen to know how the solution was obtained and what kind of mathematical methods were

used. However, if the solution can be presented graphically to the volcanologists, they will be interested to know if the solution makes sense and this solution will help in their work. Hence it is a challenge for mathematicians to be able to explain mathematical solution in a simple and friendly way.

Interpreting the solution involves identifying the meaning of the symbols (Lim *et al*., 2005). Interpreting the solution is a way to capture students' conceptual understanding of the problem and solution. It also trains students to be able to convey their understanding. A modeling project gives students practical experience of these matters.

In this project, the students were required to use the mathematical software MATLAB® to illustrate the solution graphically. The aim here was to encourage the use of technology in learning mathematics. The students had to write program for the solution using MATLAB®. Upon completing the project, each group had to give an oral presentation and a report on their findings.

A sample plot (Figure 2) of the solution (1) was given to the students as a reference, but no explanation about the plot was given to the students. Figure 2 was plotted using $Q = 10^9$kg, $X = 0$, $Y = 0$, $H = 8000$m, $U = 10$m/s, $V = 0$, $S = 1$m/s, $D_x = 800$m^2/s and $D_y = 800$m^2/s which showed that the ash was released at point $(X, Y, H) = (0,0,8000)$m and landed at about 80000m away on the ground in the *x*-direction. Students were required to interpret the location, size and shape of the deposit on the ground with respect to the parameters.
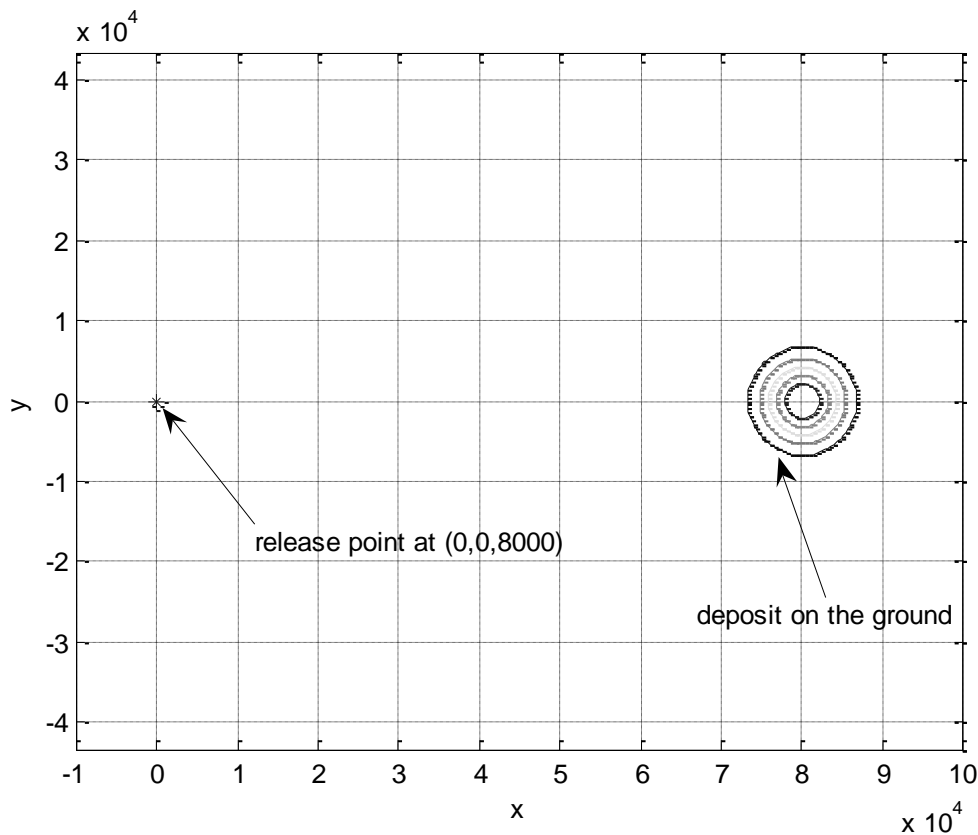

Figure 2. A sample plot of the solution.

4. Methodology

Both quantitative and qualitative methods were employed for the data collection. These were questionnaires, oral presentations, interviews and assessment. This paper only reports the oral presentations, interviews and assessment as our questionnaires were to determine the students' attitudes toward mathematics in doing the modeling. However the report about the questionnaires can be found in Lim *et al.* (2007).

4.1 Presentations

Students were grouped into eight groups, about three to four in a group and each group was required to give a presentation about their findings. The primary concern here was to assess how well the students interpreted the nine parameters (Q, X, Y, H, U, V, S, D_x and D_y) of the model. The main parameters of the model to be interpreted were H, U, V, S, D_x and D_y. Students were required to use MATLAB$^{®}$ to explore the solution. Each group had to produce at least one result graphically and explain it. Students had the freedom to vary the values of the parameters. The exploration of solution would show how students challenged themselves using MATLAB$^{®}$. We interpret the enjoyment of students as follows:

- <1 plot - The students did not or were unable to produce any result or the work was not complete. This showed that they were lost in the work or had problems in using MATLAB$^{®}$ or did not understand the work. Therefore, we concluded, it was likely that they disliked the work completely, therefore did not enjoy and were not even willing to make an effort to produce a single plot.

- 1 plot - The students produced only one result. This, we believe, showed that they either did so simply for the sake of completing the project and were not interested to find out more about the solution, or they understood the solution and did not see the point of exploring the solution further. Therefore, again, it was likely that they did not enjoy the work.

- 2 plots - The students produced two results. This, we believe, showed that they enjoyed the work and were interested to know more about the solution. Therefore, we assumed, they enjoyed the work.

- >2 plots - The students produced more than two results. We believe that this showed that they greatly enjoyed the work and were interested in discovering more about the solution.

4.2 Interviews

Interviews were conducted upon completion of the modeling project, these were also to verify and confirm the students' attitudes reflected in their questionnaires. About a fifth of the students were selected for the interview.

4.3 Assessment

Assessment was to find out if all the students had understood the model and they did participate in the project, which meant the project was not a one-man job. It was a small assessment conducted in the computer laboratory and MATLAB$^{®}$ was allowed. The students were asked to plot the deposit on the ground for the following five questions:

- What will happen to the deposit on the ground if $U = V = 0$?
- What will happen to the deposit on the ground if S is high, say higher than 1m/s?
- What will happen to the deposit on the ground if H is very low, say lower than 8000m?
- What will happen to the deposit on the ground if $D_x > D_y$?
- What will happen to the deposit on the ground if $D_x < D_y$?

5. Results and Analysis

This section summarizes the results collected from the presentations and interviews, and the analyses are discussed.

5.1 Summary of small group presentations

The students did well in their presentations and all groups produced at least two plots for the solution. Table 1 shows that all the groups had done their best to interpret at least seven parameters and Table 2 shows that they had produced at least two plots for each parameter.

Table 1. Number of parameters interpreted.

No. of parameters interpreted	7	8	9	10
No. of groups	2	3	0	3

Table 2. Number of plots produced for each parameter.

No. of plots	<1	1	2	>2
No. of groups	0	0	1	7

From the presentations, all groups appeared to enjoy the modeling work. All presentations were done using PowerPoint slides and the graphs were clearly and logically interpreted. One group even visited some websites about eruptions to get real data to plot the deposit and interpreted the cause. This group also used real data and altered some of the data to find the area covered by an eruption. All groups interpreted the parameters fairly well. They also interpreted the parameters based on their knowledge of Physics and Mathematics to explain their understanding.

Below is the brief summary about the group presentations:

Group 1 was very excited when they saw the position of deposit on the ground shifted when they varied the height of the release point. They put 'Oh God! It changes the position!' on the presentation slide. They were also the only group who experimented unrealistic values for some of the parameters and were puzzled about the results. For instance, they assigned negative sign to the settling speed S and the plot of the deposit turned upward instead of downward.

Group 2 produced the most graphs among the groups. This group was very organized. They produced different values of each parameter on the same graph to show the comparison.

Group 3 gave a very detailed conclusion at the end of the presentation about their discoveries. They also explained and interpreted their findings mathematically using ratio for comparison and were convinced about their conclusions.

Group 4 not only varied one parameter at a time, they varied two at a time for some cases to confirm their interpretation. This was actually not encouraged as they might put themselves into confusion if they were not careful.

Group 5 paid particularly attention to the parameter S and even drew a chart of S with respect to the thickness of the deposit on the ground. They found that the larger the S the thicker the deposit as the smaller the S the ash had more time to travel and so had a wider spread. However, they did not realize that they made a mistake in the calculation, when S reached more than 50 m/s (which was actually not a realistic value), the thickness of the deposit got thinner which was incorrect. This in fact did not make any sense Physically.

Group 6 was the only group who produced fewer plots for the parameters though they covered all the parameters. The group only interpreted each parameter based on two sets of values for all parameters. This group did not show any great interest in the project and the group just simply ran through the presentation.

Group 7 took the interest to further understand the parameters D_x and D_y, they interpreted that the parameters were related to U and V based on their units. The explanation was indeed correct. It was very encouraging to see the students take an effort to explore more about the parameters. In order to allow the students to explore themselves, the details of these parameters were not taught in class.

Group 8 was the only group who used the internet to search for more information about the past eruptions. The group even used the real data of an eruption on the Mount Vesuvius in Italy on the model to investigate the spread of ash fall to find out the area of the town Stabia covered by the eruption. It was very delighting to see that this group took the interest and curiosity by applying real data on the model and

interpreted the output. The group also changed some of the data when they were doubtful about the results. As in reality, real data obtained from the eruption can never be 'complete', therefore some work is always needed to estimate some of the parameters. The unit of the speeds given by the website was in 'km/hr' and the students also took the effort to convert the unit into 'm/s'. The group also tried to estimate the values of the parameters uisng the area of deposit given. This is so called 'inverse modeling', they found that in order to cover the whole town of Stabia, the release height H was one of the key factors. They varied the release height H with larger values and this was a logical move.

The Appendix summarizes some plots presented by the groups. With reference to the values of the parameters used in Figure 2, students varied the values to obtain the meaning of each parameter of the model. They also had learnt that the best way to interpret each parameter was to vary one parameter at a time.

As no information about the parameters was given to the students prior the project, they were required to program the solution to explore as many plots as they could in order to interpret each parameter of the model. The plots presented showed that the students had understood the parameters of the model thoroughly. From the presentations, we could see that they were inspired to explore the solution using MATLAB® to know more about the parameters of the model. It seen that they were not discouraged in using MATLAB® and in fact the plots produced helped them understand the model greatly given such a short time.

5.2 Interview analysis

An individual interview was conducted with six students (two males and four females). The interviews were to find out the impact of the approach of the modeling project, the use of mathematical software and their attitudes toward mathematics.

The following is the summary of the interviews with the six students:

Student 1 (female) did not see the usefulness of mathematics when she was at high school but from this project she saw its usefulness. She did not know mathematics could do such great things.

Student 2 (male) felt that the project gave him a chance to learn MATLAB®. Using MATLAB® to plot the solution allowed him to understand the parameters of the model more easily. But he found it difficult at the beginning as MATLAB® was in English. He found that mathematics was very useful in the project; it was not just simply a matter of calculation, especially the group presented the eruption on the Mount Vesuvius had shown him the importance and usefulness of mathematics.

Student 3 (female) would like to know how the solution was derived. She felt that the model was too general and it should need more parameters to give an accurate solution for the deposit. She felt that she needed to further her mathematics in order to do the modeling and she would do so after she had finished her bachelor degree.

> **Student 4** (male) found it very interesting as he was repeating the course and it would be boring to do the course all over again. He pointed that the traditional teacher simply performed before the class but this project gave the student the opportunity to perform in class. He also highlighted that from the project he learnt how to apply mathematics to real life problems and said that mathematics was a powerful tool.
>
> **Student 5** (female) said the project had helped her see things from different angles and also helped her think logically and widen her view. She learnt that there were many factors affecting the pattern of deposit on the ground. She felt that mathematics should not be just calculation or just learnt strictly from the textbook. She felt that understanding was important and interpreting helped to understand the problem.
>
> **Student 6** (female) preferred the traditional teaching in the classroom. She preferred to learn more mathematical theory than spending time on the modeling. She did not find the project interesting though she likes mathematics since young. However, she agreed that she had learnt how to interpret solution in the project.

Basically, all the six students like mathematics. The modeling project did not really affect their interest in mathematics. They felt that the project did reinforce the usefulness of mathematics. All students felt anxious at the beginning of the project as they were worried about the programming, but the feelings for some were eased after knowing what to do. All felt that the project was very 'applied' as it was a real life problem. They also found that MATLAB$^{®}$ was a useful tool for doing and learning mathematics. Though all groups were working on the same model, some groups had some interesting and different discoveries. It was a good sharing.

5.3 Assessment results

An assessment (section 4.3) about the project took place a week after the students had finished their presentations. All students got the five questions correct. From the answers given in the assessment confirmed that all students understood the parameters of the model extremely well. Moreover, many students presented the answers using the knowledge mathematics and physics. Many of them even verified their answers using MATLAB$^{®}$, this showed that they were getting comfortable or confident with the use of mathematics software.

6. Conclusion and Remarks

From the presentations, we could see that those students who explored the internet to search for the past volcanic eruption data, showed that they were curious on the reliability of the model. Students interpreted the values of the parameters based on the knowledge and theory of sciences, showed that they did not simply just accept the plot from the solution, they supported it by using the knowledge they learnt. They also constructed their explanation with mathematics to explain the logic of their findings. This showed that they did think in terms of mathematics.

Many students used MATLAB$^{®}$ to check their answers for the assessment even though the answers

were quite straightforward as they were about the general knowledge of physics. Though this was not a surprise as it is common that students usually counter check their answers using the calculator, this indicated that students were more confident and comfortable with MATLAB$^{®}$ after the project. Students also used MATLAB$^{®}$ to do mathematics homework after the project. This again showed that students were no longer afraid of using MATLAB$^{®}$.

From the interviews, majority of the interviewed students had shown positive reaction towards using MATLAB$^{®}$ for applied mathematical modeling project.

Due to the time constraints, it was a pity that we could not perform the whole modeling cycle. However, this experience and its outcomes will certainly help and encourage other colleagues at the university to use mathematical software in their mathematics courses or project since the results showed in this study were positive.

References

Clatwothy, N. J., (1989), Assessment at the Upper Secondary Level. In Blum, W., Berry, J. S., Biehler, R., Huntley, I. D. Kaiser-Messmer, G. and Profke, L. (Eds), Applications and Modeling in Learning and Teaching Mathematics (New York: John Wiley & Sons).

Lim, L. L, McKibbin, R. and Sweatman, W. L., (2005), Ashfall deposits: The effect of different source shapes. In Institute of Information and Mathematical Sciences Postgraduate Conference Proceedings, 27 October, Albany, New Zealand.

Lim, L. L., Lin, F-L and Tso, T-Y, (2007), Assessing Students' Attitudes to Mathematics: A Case Study on an Applied Mathematical Modeling Project. In C. S. Lim, S. Fatimah, S. Hajar, M. Y. Hashimah, W. L. Gan & T. Y. Hwa (Eds.), Proceedings of the Fourth East Asia Regional Conference on Mathematics Education 2007 (EARCOME4), p361-367. Penang, Malaysia: Universiti Sains Malaysia. ISBN 978-983-2700-45-6.

Lin, F. L. and Yang, K. L., (2005), Distinctive Characteristics of Mathematical Thinking in Non-modeling Friendly Environment. Teaching Mathematics and its Applications, 24 (2-3), 97-106**.**
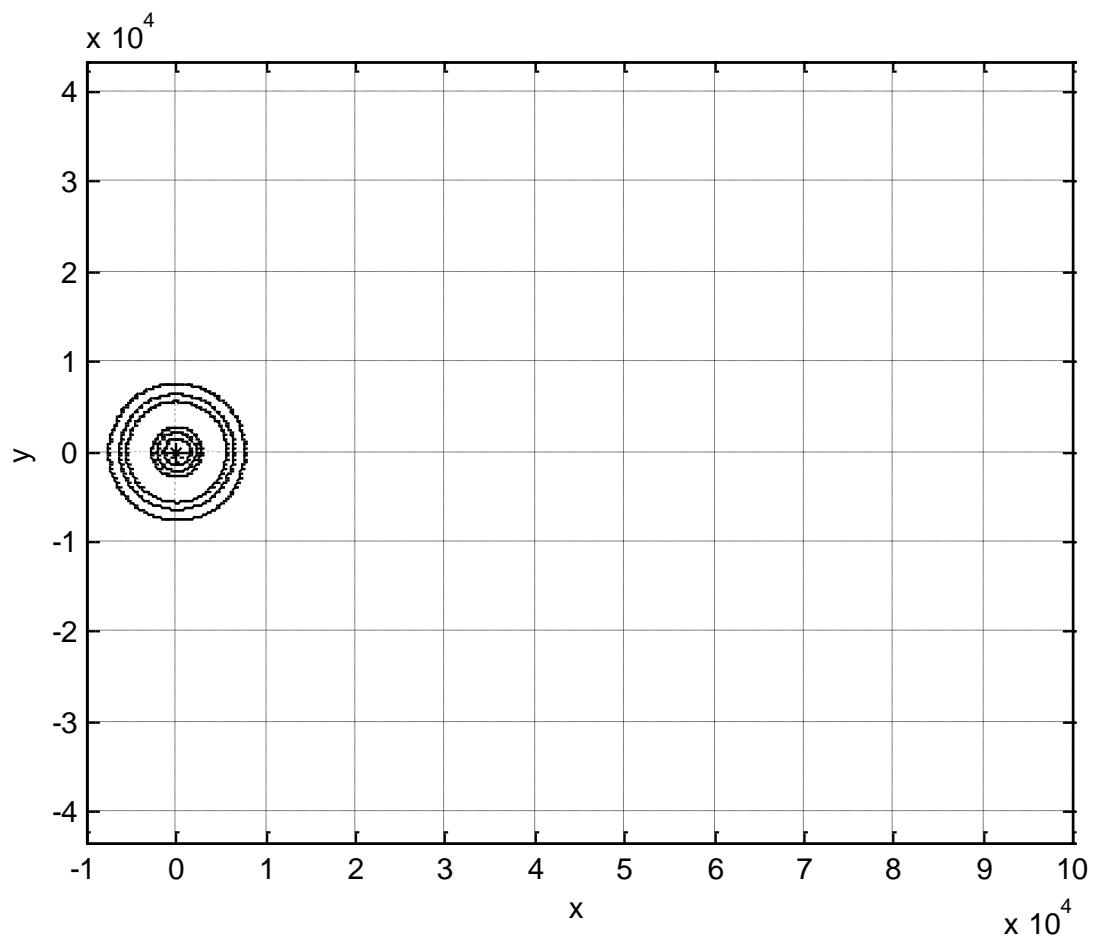
Appendix

Figure 3.

Figure 3 shows that there is no wind in both x and y directions, therefore $U = 0$ and $V = 0$. U is the wind speed in x direction and V is the wind speed in y direction. If there is no wind, the ash will automatically fall down vertically from the release point $(0,0,H)$, hence it landed and spread around the centre point $(0,0,0)$ on the ground.
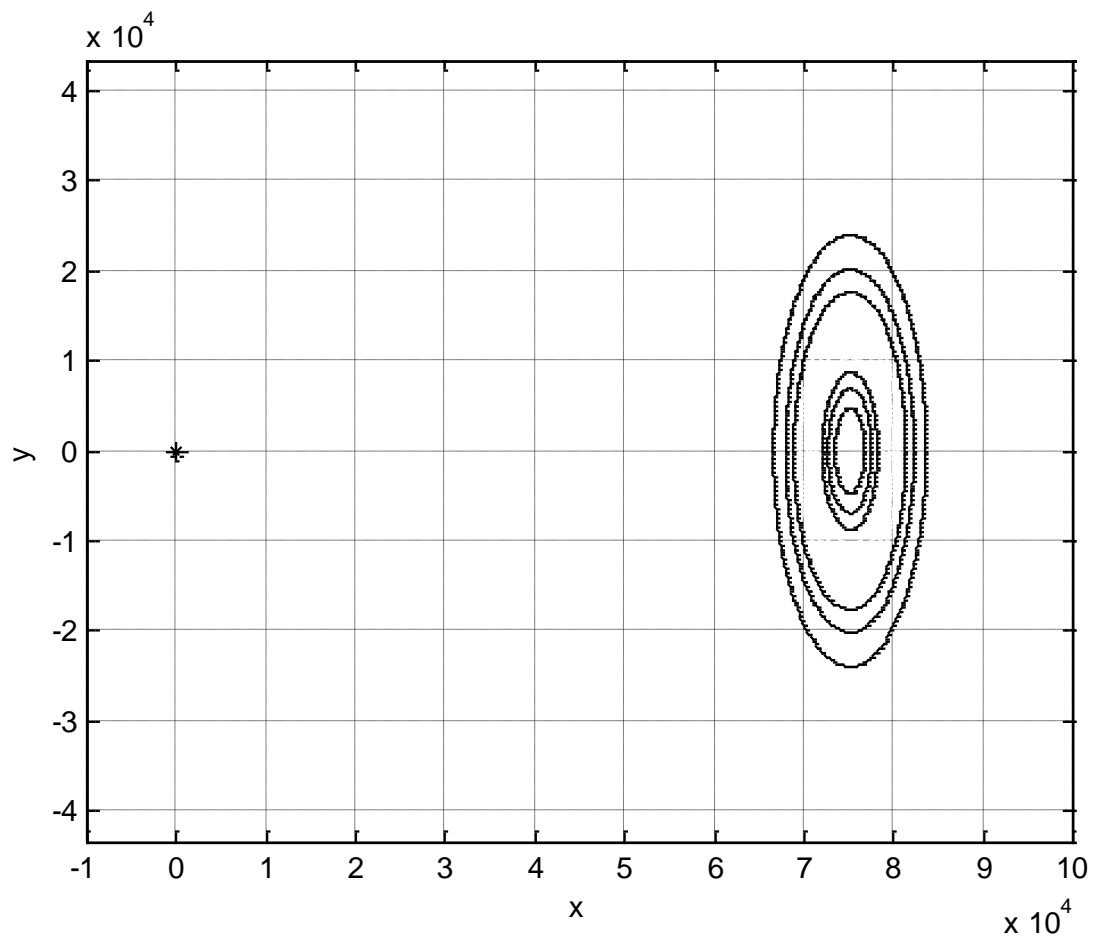
Figure 4.

Figure 4 shows that dispersion D_x is smaller than D_y as the shape is prolonged in the direction of y. D_x is the dispersion in x direction and D_y is the dispersion in y direction. If D_y is larger than D_x, the spread in y direction will be longer than in x direction, hence the shape of the deposit is prolonged in the direction of y.
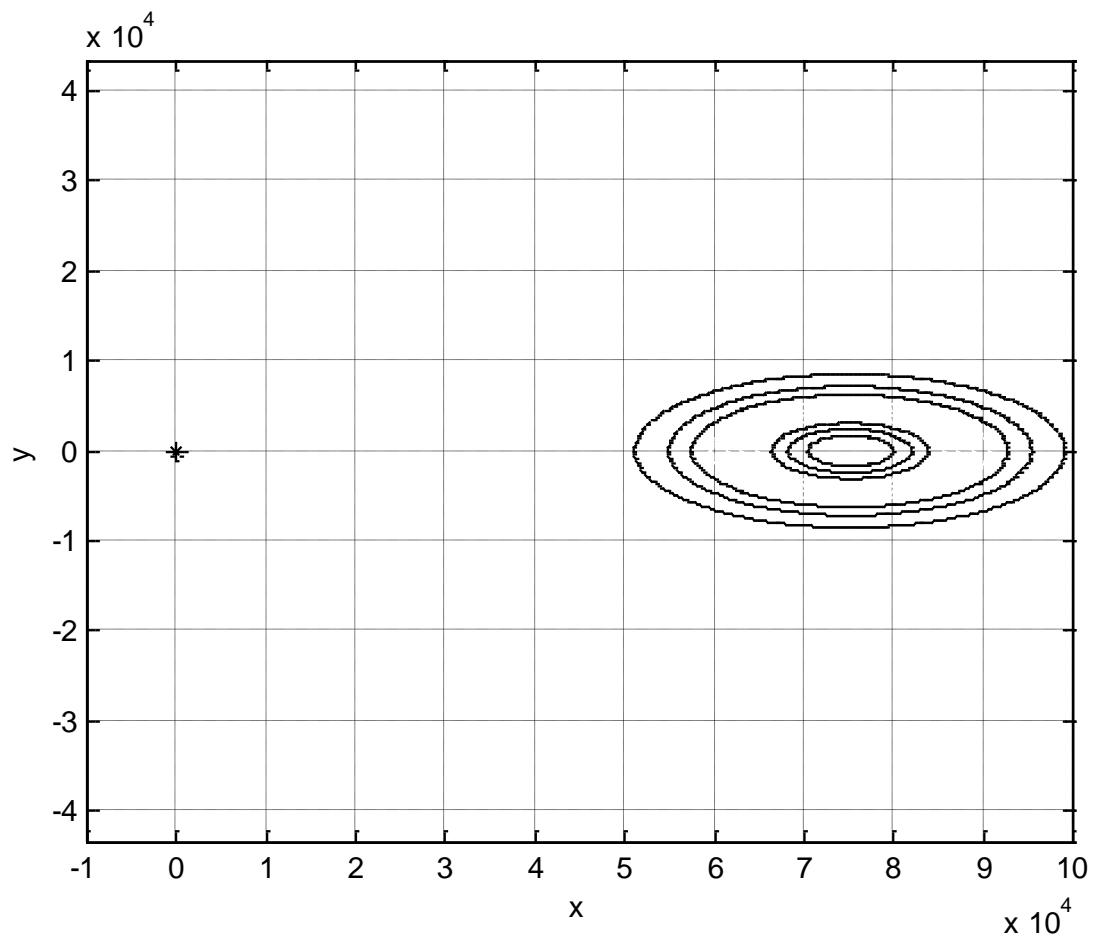
Figure 5

Figure 5 shows that dispersion D_x is larger than D_y as the shape is prolonged in the direction of x. D_x is the dispersion in x direction and D_y is the dispersion in y direction. If D_y is smaller than D_x, the spread in x direction will be longer than in y direction, hence the shape of the deposit is prolonged in the direction of x.
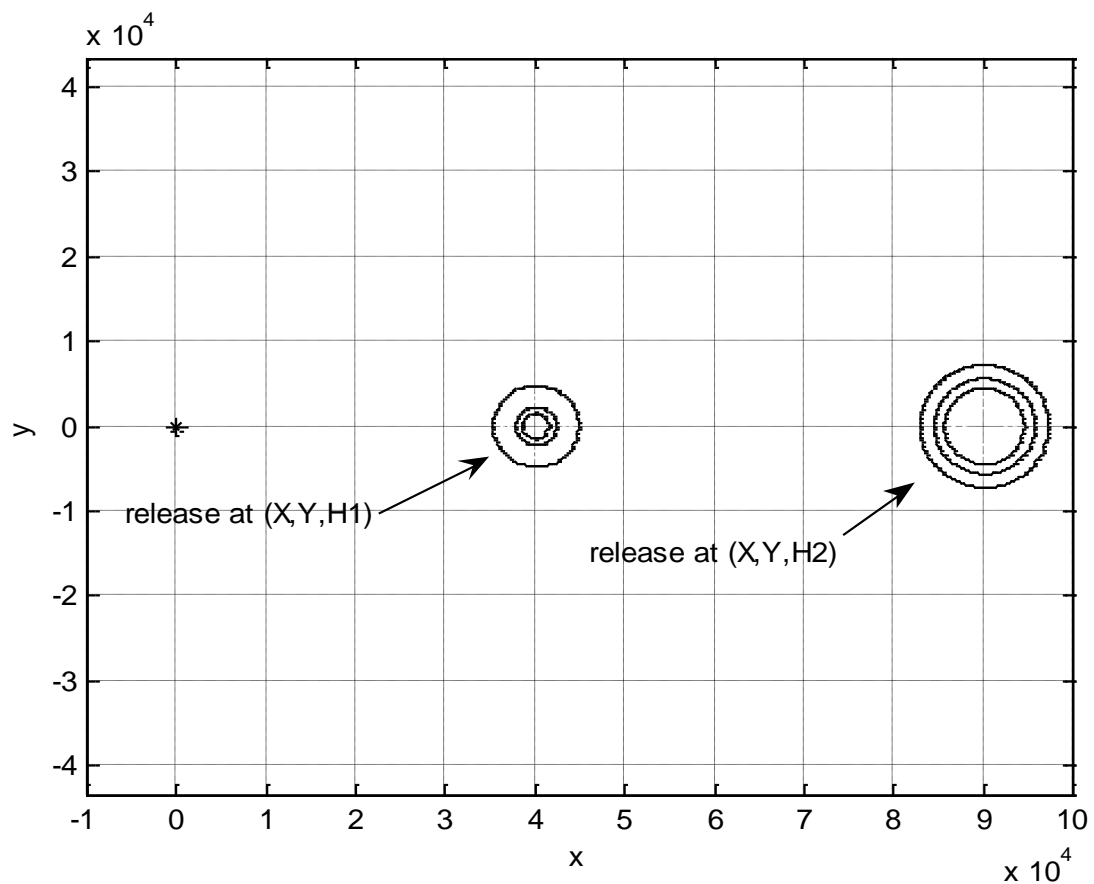
Figure 6.

Figure 6 shows that the first release at height H1 is lower than the second release at height H2, therefore it makes sense that the higher the release point, the farther the deposit as well as the wider the spread of the deposit. However, some groups found that this result might also happen even if they released at the same height. One reason could be the wind speed U in x direction of the second release was larger (or faster) than the first release, therefore it spread farther. Second reason could be the size of the particle of the second release was smaller than those at the first release. Smaller or lighter particle travels farther than the larger or heavier ones. The size of particle is determined by the falling speed S, the larger the S, the heavier the particle.

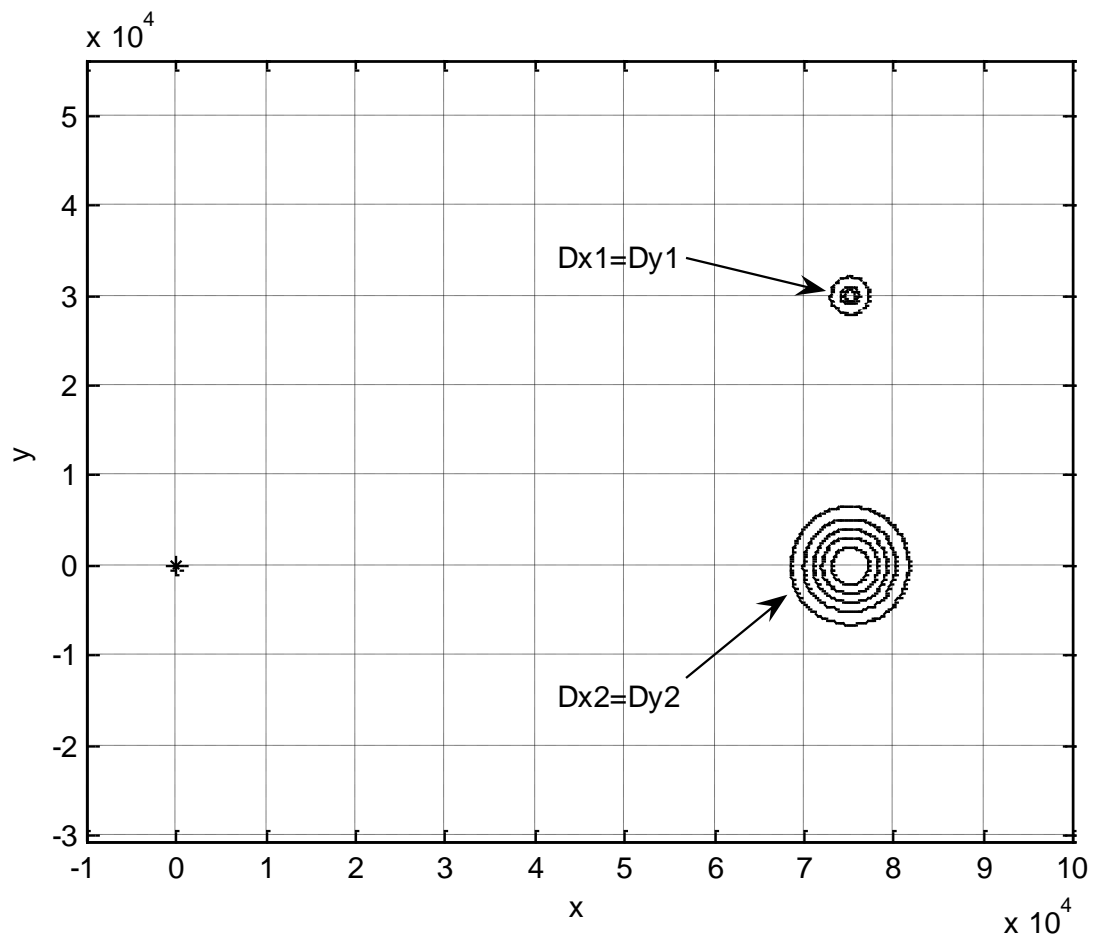Figure 7.

Figure 7 shows that the spread of the dispersion with ($Dx1 = Dy1$) is smaller than the spread of the dispersion with ($Dx2 = Dy2$), because the larger the dispersion, the wider the spread of the deposit. The shapes of the two deposits are different from those in Figures 4 and 5. The deposits in Figures 4 and 5 are oval in shape because the dispersion in x and y directions are not equal. The distance between the deposit and the release point is not measured by the value of the dispersion parameter. The dispersion parameter determines the size/area of spread.